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Core polarization and exchange effects in the inelastic scattering of tritons from $^{92,94,96}Zr$

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Abstract. A microscopic analysis of the inelastic scattering of 20 MeV tritons from ^{92}Zr, ^{94}Zr and ^{96}Zr is carried out which includes core polarization effects and exchange effects. A semi-realistic Gaussian effective interaction between the incident triton and the target nucleon is used. Inclusion of the core polarization contribution is found to decrease the necessary direct interaction strength of the collective states by at least a factor of two from those predicted by the shell-model analysis. An estimate of the relative importance of exchange effects is made using a simple pseudopotential. Exchange terms are found to be important for most of the single-particle shell-model transitions studied.

1. Introduction

In recent years efforts have been made to interpret the results of certain inelastic composite particle experiments in terms of a microscopic description of the states involved (Bernstein 1969, Glendenning and Veneroni 1966, Glendenning 1969, Madsen 1966, Park and Satchler 1970, Satchler 1965, Satchler 1966). In this microscopic approach the initial and final states involved in the transition are assumed to be reasonably good shell-model states. This model also assumes that the interaction which induces the transition can be expressed as a sum of the two-body effective interactions between the projectile and each target nucleon. The interaction is obtained by folding a semi-realistic nucleon–nucleon interaction into the density distribution of the projectile (Bernstein 1969, Glendenning and Veneroni 1966, Madsen 1966, Park and Satchler 1970, Park 1973, Satchler 1966).

The inclusion of the effects due to core polarization involves a renormalization of the effective interaction induced by a virtual excitation, usually referred to as polarization of the core by the projectile. This renormalization of the interaction has been studied recently by Park and Satchler (Park and Satchler 1970, Park 1973) for tritons and for ^{3}He particles which were inelastically scattered from ^{90}Zr. The possibility of nucleon exchange between the target and the projectile may arise both from the antisymmetrization of the projectile and target wavefunctions as well as from the exchange character of two-body forces. The estimate of its importance for the inelastic scattering of ^{3}He particles from ^{40}Ca has been made (Park and Satchler 1970) recently using a zero-range pseudopotential for the exchange terms (Schaeffer 1970). It was found, as in the case of proton scattering (Love *et al* 1969, Love and Satchler 1970, Petrovich *et al* 1969, Philpott and Pinkston 1969, Schaeffer 1969), that the magnitude of cross sections increases appreciably with the inclusion of exchange effects.

The stable zirconium isotopes are interesting nuclei for nuclear spectroscopy since ^{90}Zr lies at the $N = 50$ major neutron shell closure and the added neutrons occupy primarily the $2d_{5/2}$ shell-model orbital until the orbital is essentially full for ^{96}Zr (Cohen and Chubinsky 1963). Most of the low-lying levels of the stable zirconium isotopes have been described in terms of the shell model with configuration mixing. Inelastic scattering (Flynn *et al* 1970, Stautberg and Kraushaar 1966) as well as transfer reactions have been used to study the level structure of the ^{92}Zr, ^{94}Zr and ^{96}Zr nuclei (Flynn *et al* 1974, Ball *et al* 1971).

The primary objectives of the present work were the systematic investigations of both the effects of core polarization and the effects of nucleon exchange between target and projectile in the inelastic scattering of tritons from the Zr isotopes, ^{92}Zr, ^{94}Zr and ^{96}Zr. In the present analysis inelastic scattering data of Flynn *et al* (1970) with 20 MeV tritons have been used.

2. Model

2.1. *Effective interaction*

Inelastic scattering cross sections depend on a nuclear matrix element which contains both the wavefunctions of the nuclear states involved and the effective interaction between the projectile and the target nucleon. It is, therefore, necessary to have a correct effective interaction before one can use inelastic scattering as a tool for nuclear spectroscopy. For inelastic proton scattering several attempts (Love and Satchler 1970, Petrovich *et al* 1969, Schaeffer 1969) have been made to obtain 'realistic' effective interactions from the free two-body forces. For example, Love and Satchler (1970) have shown that a Gaussian interacction which fits low-energy nucleon–nucleon scattering gives results close to the long-range part of the Hamada–Johnston potential (Hamada and Johnston 1962), provided its strength is renormalized. Since this approach has been relatively successful, the same model may be applied to the scattering of tritons. The effective triton–nucleon interaction which we have used is obtained by folding a realistic nucleon–nucleon interaction into the density distribution of the triton (Park and Satchler 1970, Park 1973). For convenience, a Gaussian form is used both for the two-nucleon interaction and for the triton density distribution. The interaction thus obtained may be called 'realistic' in the sense that the nucleon–nucleon interaction used (Wong and Wong 1967) reproduces low-energy nucleon–nucleon scattering and is approximately equivalent to a reaction matrix based upon the Hamada–Johnston or Kallio–Kolltveit potentials (Kallio and Kolltveit 1965).

We have used, therefore, in the present calculations a spin-independent effective interaction of a Gaussian form given by:

$$V_{\mathrm{tn}} = 22{\cdot}5 \exp(-0{\cdot}20\, r_{\mathrm{tn}}^2), \tag{1}$$

where r_{tn} is the distance between the target nucleon and the centre of mass of the incident triton and V_{tn} is in MeV. This effective triton–nucleon interaction has neglected the possibility of break-up of the triton.

2.2. *Distorted wave formalism*

The transition amplitude for the (t, t') process in the distorted wave (DW) approximation is given by:

$$T_{\mathrm{if}} = \int \mathrm{d}\boldsymbol{r}_{\mathrm{i}}\, \mathrm{d}\boldsymbol{r}_{\mathrm{f}}\, \chi_{\mathrm{f}}^{*(-)}(\boldsymbol{k}_{\mathrm{f}}, \boldsymbol{r}_{\mathrm{f}}) \langle \phi_{\mathrm{f}} | V | \phi_{\mathrm{i}} \rangle \chi_{\mathrm{i}}^{(+)}(\boldsymbol{k}_{\mathrm{i}}, \boldsymbol{r}_{\mathrm{i}}). \tag{2}$$

An incident triton, represented by a distorted wave $\chi_{\mathrm{i}}^{(+)}$, interacts with a valence nucleon in the bound state ϕ_{i}, thereby raising the nucleon to an excited bound state ϕ_{f}. The scattered triton is represented by an outgoing distorted wave $\chi_{\mathrm{f}}^{(-)}$. The theoretical cross section $\sigma_L(\theta)$ is proportional to $|T_{\mathrm{if}}|^2$ and the experimental cross section $(\mathrm{d}\sigma/\mathrm{d}\Omega)_{\mathrm{exp}}$ is related to the theoretical cross section $\sigma_L(\theta)$ (Johnson *et al* 1966, Satchler 1964) by:

$$\left(\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}\right)_{\mathrm{exp}} = (2J+1)|V|^2 M_L^2 \sigma_L(\theta), \tag{3}$$

where J represents the total angular momentum transferred, V, the depth of the interaction between the incident triton and the target nucleon. The spin-angle reduced matrix M_L is given by (Johnson *et al* 1966, Satchler 1964):

$$M_L = \langle \mathrm{f} \| \mathrm{i}\, Y_L(\theta, \phi) \| \mathrm{i} \rangle. \tag{4}$$

The square of the radial form factor, $I_L(r)$, defined by

$$I_L(r) = \int u_2(r_{\mathrm{i}}) u_1(r_{\mathrm{i}}) g_L(r_{\mathrm{i}}, r)\, \mathrm{d}^3 r_{\mathrm{i}} \tag{5}$$

determines the radial dependence of the theoretical cross section. Here, u_1 and u_2 represent the radial wavefunctions for the valence nucleon in the initial and final states, respectively, and g_L is the radial part of the multipole expansion of the interaction (equation (1)) between the triton and the valence nucleon. The radial form factors are numerically calculated using the computer code ATHENA (Chwieroth *et al* 1969). The values of the binding energies for the various transitions were obtained from a standard reference (Garvey *et al* 1969). The distorted waves $\chi_{\mathrm{i}}^{(+)}$ and $\chi_{\mathrm{f}}^{(-)}$ are obtained as a solution to the Schrödinger equation containing an optical model potential which fits the elastic scattering data. The differential cross sections are calculated by the computer code JULIE (R M Drisko 1965, unpublished). The optical potential used in the present analysis has the Woods–Saxon form with volume absorption:

$$V(r) = V_{\mathrm{C}}(r) - V(1+\mathrm{e}^{x})^{-1} - \mathrm{i}W(1+\mathrm{e}^{x'})^{-1}, \tag{6}$$

with

$$x = (r - r_0 A^{1/3})/a, \qquad x' = (r - r_0' A^{1/3})/a',$$

and $V_{\mathrm{C}}(r)$ is the Coulomb potential due to a uniformly charged sphere of radius $r_{\mathrm{C}}A^{1/3}$. We have used the optical potential parameters determined by Flynn *et al* (1970) from their elastic scattering data. These parameters are listed in table 1.

Table 1. Optical model parameters used (from Flynn *et al* 1970).

Nucleus	r_{C} (fm)	V (MeV)	W (MeV)	r_0 (fm)	r_0' (fm)	a (fm)	a' (fm)
^{92}Zr	1·25	159·1	25·0	1·24	1·401	0·670	0·809
^{94}Zr	1·25	153·4	18·8	1·24	1·489	0·681	0·787
^{96}Zr	1·25	154·0	18·6	1·24	1·391	0·672	0·990

2.3. Core polarization

Core polarization effects have been used successfully in proton inelastic scattering (Love and Satchler 1967, 1970, Agassi and Schaeffer 1970) to explain large cross sections for states which can be described partly as single particle excitations. The large cross sections indicate that some other interaction occurs besides the single-particle excitation. Core polarization is the renormalization of the effective interaction between the projectile and the extra-core nucleon. That is, it is the additional contribution to the cross section due to indirect excitation of the valence nucleon via a virtual excitation or, equivalently, polarization of the core by the projectile. In this process, the core is de-excited by exciting the valence nucleon. The form of the core contribution is obtained by assuming a collective model for the core of closed shells and the strength is obtained by applying this model to the relevant electromagnetic transition for which the direct or non-core part of the interaction is known. The effect of core polarization may be included using the form factor given by (Love and Satchler 1967, 1970)

$$F(r) = V_{\mathrm{dir}} I_{\mathrm{dir}} + V_{\mathrm{core}} I_{\mathrm{core}}, \tag{7}$$

where

$$V_{\mathrm{core}} I_{\mathrm{core}} = \frac{y_L \langle k_{\mathrm{n}} \rangle}{4\pi} \left(V_{\mathrm{opt}} \frac{R}{a} \frac{\mathrm{d}f}{\mathrm{d}x} + \mathrm{i} W_{\mathrm{opt}} \frac{R'}{a'} \frac{\mathrm{d}f'}{\mathrm{d}x'} \right), \tag{8}$$

with

$$f = (1 + \mathrm{e}^x)^{-1}, \qquad x = (r-R)/a, x' = (r-R')/a', \qquad R = r_0 A^{1/3}. \tag{9}$$

Here, $\langle k_{\mathrm{n}} \rangle = \langle n_2 l_2 j_2 | k_{\mathrm{n}} | n_1 l_1 j_1 \rangle$ denotes the radial integral for the valence nucleon. The parameter $A_L = y_L \langle k_{\mathrm{n}} \rangle$ is a measure of the strength of core polarization coupling and is characteristic of the transition and is independent of the projectile as well as of the bombarding energy. Its value may be extracted from the measured value of the electromagnetic transition rate $B(\mathrm{E}L)$ (Satchler and Love 1971).

2.4. Exchange

For the case of proton inelastic scattering it has been shown by Petrovich *et al* (1969) and others (Schaeffer 1969, Love *et al* 1969, Love and Satchler 1970) that exchange of the incident proton with a target nucleon can be very important, particularly for transitions of high multipolarity. The possibility of the exchange may arise both from exchange forces and from antisymmetrization of the projectile and target nucleon wavefunctions. The antisymmetrization required by the Pauli principle can give rise to exchange potentials as shown, for example, in the case of heavy ion scattering (Park *et al* 1972). The importance of including exchange effects when generating transition potentials for inelastic scattering from a folding model was indicated recently by Satchler (1972). An estimate of the exchange effects for composite particles has been made by Schaeffer (1970) based upon an approximation introduced for proton scattering (Petrovich *et al* 1969). We have included the exchange term in the usual way (Schaeffer 1970) using a zero-range pseudopotential to study the relative importance of the nucleon exchange between the triton and the Zr nuclei. We represent the given pseudopotential with:

$$V_{\mathrm{E}} = -V_{\mathrm{EO}} \exp(-r^2/\lambda^2). \tag{10}$$

The strength V_{EO} depends upon the bombarding energy of the triton as given by (Schaeffer 1970)

$$V_{EO} = V_{E}(\mu/\lambda)^3 \exp(-k^2\mu^2/4), \tag{11}$$

where k denotes the wavenumber of the incident nucleon with the same energy as the triton, and μ represents the nucleon–nucleon range. Thus, V_{EO} decreases for higher bombarding energies. The interaction also depends upon the type of exchange which takes place between the target and the projectile. We have considered only space exchange, that is, Serber exchange and used the values (Schaeffer 1970) $V_{EO} = 90$ MeV, $\lambda = 1{\cdot}37$ fm and $\mu = 1{\cdot}78$ fm for 20 MeV tritons.

3. Results and discussion

The transitions we have studied and the corresponding final-state neutron configurations we have used are shown in table 2. We have assumed the same neutron configurations as Flynn *et al* (1970) for the final states.

Table 2. Strengths of effective Gaussian interaction with the range parameter $\gamma = 0{\cdot}20\ \mathrm{fm}^{-2}$ for excitation of low-lying states of ^{92}Zr, ^{94}Zr and ^{96}Zr. For comparison the Yukawa strengths with $\alpha = 1{\cdot}0\ \mathrm{fm}^{-1}$ deduced by Flynn *et al* (1970) are also listed.

Assumed neutron configuration	L	^{92}Zr E_x (MeV)	V_{0G} (MeV)	V_{0y} (MeV)	^{94}Zr E_x (MeV)	V_{0G} (MeV)	V_{0y} (MeV)	^{96}Zr E_x (MeV)	V_{0G} (MeV)	V_{0y} (MeV)
$(2d_{5/2})^2_2$	2	0·93	101	700	0·92	69	620			
$(2d_{5/2})^2_4$	4	1·49	53	320	1·47	31	290			
$(2d^{-1}_{5/2}\,3s_{1/2})_2$	2	2·85	45	220	2·34	46	210	1·74	27	240
$(2d^{-1}_{5/2}\,2d_{3/2})_2$	2							3·20	25	190
$(2d^{-1}_{5/2}\,2d_{3/2})_4$	4	3·14	24	290	3·35	33	350	3·13	33	220
$(2d^{-1}_{5/2}\,1g_{7/2})_2$	2							3·76	50	302
$(2d^{-1}_{5/2}\,1g_{7/2})_4$	4							3·74	35	212
$(2d^{-1}_{5/2}\,1g_{7/2})_6$	6							3·63	24	150

In order to examine which transitions can be described simply as direct single-particle transitions we have calculated, at first, the differential cross sections using direct, shell-model form factors with the 'realistic' Gaussian interaction of equation (1) and compared the results with the cross sections measured by Flynn *et al* (1970). The effective Gaussian interaction strengths needed to account for the magnitude of the measured cross section using direct, shell-model form factors are listed in table 2. Effective Yukawa interaction strengths deduced by Flynn *et al* (1970) from a shell-model calculation without including core polarization effects are also listed for comparison. They obtained a triton interaction strength about three times that of the proton deduced in the p, p′ reactions on the Zr isotopes.

The larger effective interaction strengths needed for the transitions suggest that processes other than direct single-particle shell-model excitations may be involved. We have considered two possibilities: (i) an exchange of nucleons between the projectile and the target nucleus; and (ii) the core polarization, namely, an indirect excitation of the valence neutron via a virtual excitation, polarization, of the core by the projectile.

We have made an estimate of the relative importance of exchange effects using the model described in § 2.4. Inclusion of the exchange contribution has improved the agreement between the calculated and measured cross sections considerably for most of the transitions. The importance of exchange effects is illustrated in figure 1 for the

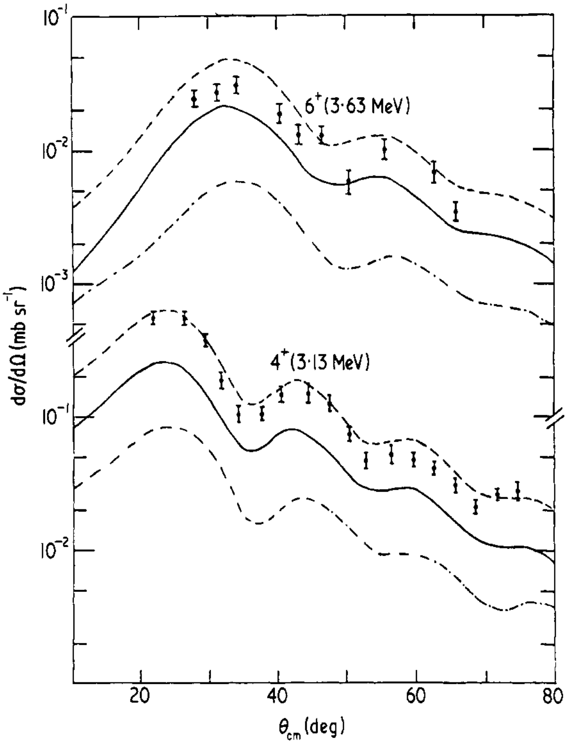

Figure 1. Comparison of the calculated and measured cross sections for the transitions leading to the first 4^+ and 6^+ states in ^{96}Zr at 3·13 and 3·63 MeV respectively. Exchange effects are illustrated. The full curves represent the cross sections with direct shell-model form factors, the chain curves, with exchange terms only, and the broken curves, with the coherent sums of the direct and the exchange contributions.

first $L = 4$ and the $L = 6$ states in ^{96}Zr at 3·13 and 3·63 MeV, respectively. The chain curves represent the exchange cross sections and the full curves the direct cross sections. The cross sections as calculated from the coherent sum of the direct and the exchange interactions are represented by the broken curves. There are, however, some transitions in which the inclusion of the possibility of exchange overestimates the experimental cross sections. We found three such cases. These are the transitions leading to the first two 2^+ states in ^{96}Zr and the second 4^+ state of ^{92}Zr. Figure 2 illustrates the case for the first 2^+ states in ^{96}Zr. Possible reasons for the overestimation of the first 2^+ states in ^{96}Zr could be that the exchange effects are found to be least when the angular momentum transferred is smallest and that the $2d_{5/2}$ neutron

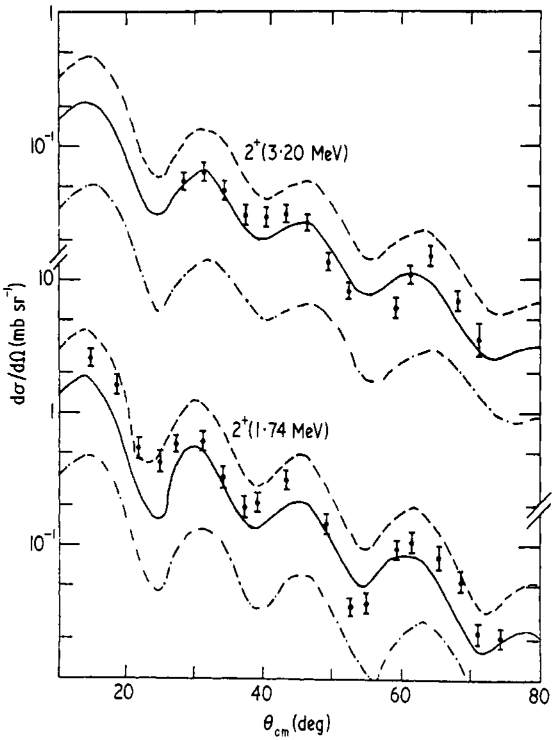

Figure 2. Comparison of the calculated and the measured cross sections for the direct single-particle transitions leading to the 2^+ states at 1·74 and 3·20 MeV in ^{96}Zr. The various lines represent the similar cases as discussed in the caption of figure 1.

shell is filled for ^{96}Zr and hence the binding energy of the neutrons in this orbit is larger than for ^{92}Zr or ^{94}Zr.

The results of our calculations to determine the relative importance of the exchange effects are summarized in table 3. We note that the ratios of the exchange cross section σ_E to the direct cross section σ_D are less than $\frac{1}{2}$ for all states in all nuclei. However, the ratio σ_{D+E}/σ_D of the cross section which includes both the direct and the exchange contributions to that of the direct cross section only is greater than two. This indicates that the contribution from nucleon exchange is important for most transitions.

There are some transitions for which the inclusion of the possibility for neutron exchange is still not sufficient to predict the experimental cross sections. The first $L = 2$ transitions in the ^{92}Zr and the ^{94}Zr isotopes are such cases. This suggests that these states are collective in nature, as is the case with the first $L = 2$ state in ^{90}Zr at 2·18 MeV. Since the inclusion of the core polarization effects has successfully described the ^{90}Zr(t, t′) ^{90}Zr* (2·18 MeV) reaction induced by 20 MeV tritons (Park and Satchler 1970), we have considered for the ^{92}Zr and ^{94}Zr isotopes the same possibility of core polarization for the transitions. For the first 2^+ transition in ^{92}Zr and in ^{94}Zr experimental values for the electromagnetic transition rates B(E2) are known. Therefore, the core coupling parameters for quadrupole transitions $A_2 = y_2\langle k_n\rangle$ can be obtained in the usual way (Park 1973) by applying the collective vibrational model for the core to the relevant electromagnetic transition. There are, however, some uncertainties, in the measured B(E2) transition rates (Galperin *et al* 1969, Gangrskii and Lemberg

Table 3. Relative importance of exchange effects.

	State (MeV)	Transferred angular momentum	σ_E/σ_D	σ_{D+E}/σ_D
^{92}Zr	0·93	2^+	0·2232	2·10
	1·49	4^+	0·3268	2·37
	2·85	2^+	0·2725	2·26
	3·14	4^+	0·3546	2·45
^{94}Zr	0·92	2^+	0·2427	2·15
	1·47	4^+	0·3484	2·43
	2·34	2^+	0·2958	2·34
	3·35	4^+	0·3660	2·50
^{96}Zr	1·74	2^+	0·2724	2·25
	3·13	4^+	0·3496	2·43
	3·20	2^+	0·2604	2·19
	3·63	6^+	0·4681	2·21
	3·74	4^+	0·2462	2·12
	3·76	2^+	0·1685	1·91

1965, Grinberg *et al* 1960, Stelson and Grodzins 1965). The A_2 values we obtained for the first 2^+ states in ^{92}Zr, ^{94}Zr and ^{96}Zr are 0·28, 0·20 and 0·08 respectively. Subsequently, these values were used in our calculations. From the magnitude of the A_2 values, it is seen that the core polarization effects for the 2^+ state in ^{96}Zr is negligibly small. This is the reason, as noted earlier, why the description of this state as a direct single-particle transition predicted the experimental cross section. The calculated cross sections including core polarization effects with coupling strengths A_2 which are determined independently from experimental data are compared with the measured cross sections for the first 2^+ states in ^{92}Zr and ^{94}Zr in figure 3. The lower full curves represent the predicted cross section calculated using a semi-realistic direct interaction ($V_0 = 22{\cdot}5$ MeV, $\gamma = 0{\cdot}20\ \mathrm{fm}^{-2}$) between the triton and the valence neutrons. The cross sections calculated from employing only a virtual, complex core excitation are represented by the chain curves. The broken curves represent the cross sections determined by coherently summing the direct shell-model and the core polarization contributions. We note that the contribution of the direct part alone is quite small and that the core polarization accounts for a very large portion of the measured cross section. It is seen that the calculated cross sections including both the direct and the core polarization contributions predict fairly well the observed cross sections. Separate calculations were carried out using only a real or an imaginary part of the form factor due to core polarization to examine if the core interaction is largely real or imaginary. As is the case of ^{90}Zr(t, t') ^{90}Zr*(2·18 MeV, 2^+) analysed by Park (1973) earlier, it is seen in figure 3 that the contributions from the real and imaginary parts of the core coupling for ^{92}Zr, denoted by the upper full curve and the dotted curves respectively, are almost equal.

It is possible that some of the transitions which may be described as single-particle transitions with the possibility of exchange may be explained equally by including core polarization effects with no explicit exchange contribution. For example, we found that the second 2^+ states in ^{92}Zr at 2·85 MeV and in ^{94}Zr at 2·34 MeV could be described just as well without exchange contributions if core polarization with core

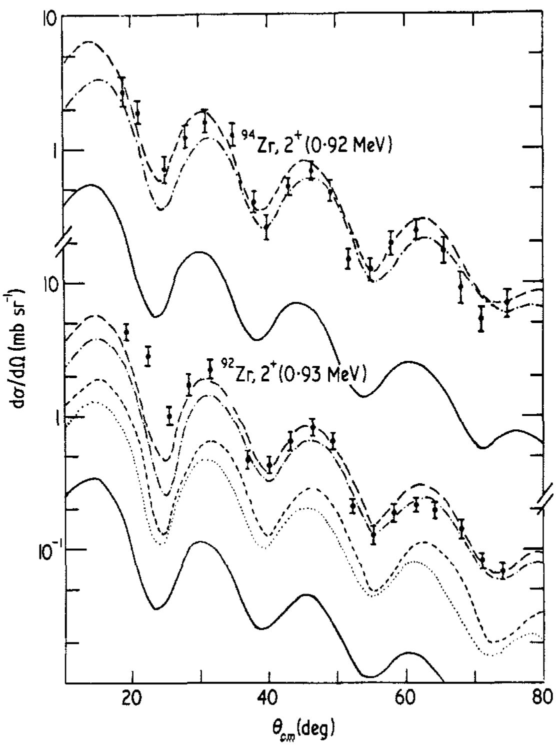

Figure 3. Comparison of the calculated and the measured cross sections for the transitions leading to the first 2^+ states in ^{92}Zr and ^{94}Zr. Effects of core polarization using the coupling strengths deduced independently from experimental *B*(E2) values are illustrated. The full curves represent the cross sections with direct shell-model form factors, the chain curves, the core polarization contribution only, and the upper broken curve, with the coherent sums of the direct and the core polarization contributions. In addition in ^{92}Zr the lower broken curves and the dotted curve represent the cross sections to real and imaginary core polarization contributions respectively.

coupling strengths of 0·13 and 0·12, respectively, were included. The strengths we found for the 2^+ states without exchange and without core polarization were as listed in table 1 (46 and 45 MeV, respectively), which indicated that some collective strength was present.

Similarly, we determined the core coupling strengths needed to fit the first 4^+ states in ^{92}Zr at 1·49 MeV and ^{94}Zr at 1·47 MeV without exchange. These were found to be $A_4 = 0{\cdot}08$ for ^{92}Zr and $A_4 = 0{\cdot}05$ for ^{94}Zr. Our study showed that the contribution from core polarization decreased with increasing binding energy and that the core contributes less to the transitions with higher angular momentum transfer. However, for the cases where no core coupling parameters A_L are currently available, these results may not be as meaningful as the ones containing exchange.

There is one additional concern with the ^{96}Zr isotope which does not occur with the ^{92}Zr and ^{94}Zr isotopes. Since the $(2d_{5/2})$ neutron shell is filled in ^{96}Zr, the possibility of proton excitation, as well as neutron excitation occurs, analogous to the ^{90}Zr case (Flynn *et al* 1968). Proton excitations in ^{90}Zr by tritons and by helions were studied by Park and Satchler (1970), and Park (1973). The ground state configuration for the ^{96}Zr isotope is given by:

$$\langle \mathrm{i}| = a|\nu 2\mathrm{d}_{5/2}\rangle_0^6|\pi 2\mathrm{p}_{1/2}\rangle_0^2 + b|\nu 2\mathrm{d}_{5/2}\rangle_0^6|\pi 1\mathrm{g}_{9/2}\rangle_0^2. \qquad (12)$$

The first 2^+ state in ^{96}Zr, for example, may be due to recoupling of $1g_{9/2}^2$ protons (Gloeckner 1972) just as the first 2^+ state in ^{90}Zr is assumed to be (Cates *et al* 1969). The cross section due to the direct interaction for the proton transition will be proportional to b^2, which has the value of 0·135 for ^{96}Zr (Preedom *et al* 1968). This is to be compared with that for a neutron transition which is proportional to $a^2 = 0{\cdot}865$ (Preedom *et al* 1968). Therefore, the direct part of the interaction for a proton transition is considerably less than for a neutron excitation. We found that the core polarization effects are important for proton excitations in ^{96}Zr and can account for a large part of the measured cross sections as found for ^{90}Zr.

As stated previously, the core coupling parameter we calculated for the neutron state, was found to be small. The core coupling parameter A_L not only depends upon the B(E2) value but it also depends inversely upon the M_L value:

$$A_L = \frac{4\pi}{3}\left(\frac{2J_{\mathrm{i}}+1}{2J_{\mathrm{f}}+1}\right)^{1/2}\frac{(B_{\mathrm{i}\to\mathrm{f}}(\mathrm{E}L)^{1/2}}{Z_{\mathrm{C}}eR_{\mathrm{C}}^L M_L} - \begin{cases} 0 & \text{(for neutron)} \\ \dfrac{4\pi}{3}\dfrac{\langle r^L\rangle}{Z_{\mathrm{C}}R_{\mathrm{C}}^L} & \text{(for proton).} \end{cases}$$

Therefore, whether the first 2^+ state is a neutron excitation $(2d_{5/2}^{-1}\,3s_{1/2}^1)$ with $M_L = 0{\cdot}4886a$ or a recoupling of protons to $(1g_{9/2})_2^2$ with $M_L = 0{\cdot}3106b$ will strongly affect the value of the core coupling parameter A_L (Ball *et al* 1968). Specifically, we find that $A_L = 0{\cdot}272$ for the $(1g_{9/2})_2^2$ proton excitation and $A_L = 0{\cdot}08$ for the $(2d_{5/2}^{-1}\,3s_{1/2}^1)$ neutron excitation. However, there is about a 40% uncertainty in the measured B(E2) value for ^{96}Zr. In addition, the mean value for r^2 has not been explicitly determined for ^{96}Zr. The two uncertainties coupled with the availability of only one measurement of the B(E2) value for ^{96}Zr, make it difficult to state conclusively at the present time whether the first 2^+ state at 1·74 MeV in ^{96}Zr is a proton state, or as previously assumed, a neutron state.

4. Conclusions

A satisfactory microscopic description of the triton inelastic scattering from ^{92}Zr, ^{94}Zr and ^{96}Zr can be given when the exchange effects or core polarization effects are taken into account. Among the transitions to the low-lying states we have examined, only the first two 2^+ states in ^{96}Zr at 1·74 and 3·20 MeV can be described as a single-particle transition involving a direct interaction of the projectile with the valence nucleons. On the other hand, the measured cross sections leading to the excitation of the first 2^+ states in ^{92}Zr and ^{94}Zr only can be accounted for when the contribution from core polarization is included. Most other states can be explained as single-particle transitions provided that exchange effects or core polarization effects are taken into consideration.

We have used a semi-realistic interaction, Gaussian in form with 22·5 MeV strength and 2·2 fm range ($\gamma = 0{\cdot}20\,\mathrm{fm}^{-2}$), which is obtained by folding a 'realistic' nucleon–nucleon interaction into the triton density distribution. Core polarization effects are taken into account by using a simple collective model for the core with the coupling strengths obtained independently from the measured B(EL) values. The core contribution is found to be important for the collective first 2^+ states in ^{92}Zr and ^{94}Zr, as was the case for ^{90}Zr. Estimates of the relative importance of the exchange effects have been made using a simple pseudopotential (Schaeffer 1970) for the exchange term. It is found, as in the case of proton scattering, that the magnitudes of the theoretical cross sections are enhanced appreciably, but the shape of angular distributions is affected little. The relative importance of exchange effects in the triton inelastic scattering also increases smoothly with increasing angular momentum transfer L but does not seem to show any strong dependence on binding energy of the valence nucleons.

Recently, Satchler (1970, 1971) has suggested that the microscopic effective interaction may contain an important imaginary component and has given a recipe for determining it from the imaginary part of the optical model potential. While the triton is bound more strongly than the deuteron, the possibility of break-up still exists and inclusion of the imaginary effective interaction may improve the fit of the theoretical cross section to the experimental data.

Since the neutrons in the triton are paired to $S = 0$, there may be a possibility of two-neutron exchange as well as single-particle exchange. We have not taken into consideration the possibility of two-particle excitation in the present calculation. It is also an interesting problem to study the effects of non-central forces, such as tensor and spin–orbit interactions, on the inelastic scattering of composite particles.

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